P12CH6

TOPIC

6

High Energy Physics

6.1 PARTICLE PHYSICS

Particle physics or **high energy physics** is the study of fundamental particles and forces that constitute matter and radiation. The fundamental particles in the universe are classified in the Standard Model as fermions (matter particles) and bosons (force-carrying particles). There are three generations of fermions, but ordinary matter is made only from the first fermion generation. The first generation consists of up and down quarks which form protons and neutrons, and electrons and electron neutrinos. The three fundamental interactions known to be mediated by bosons are electromagnetism, the weak interaction, and the strong interaction.

Quarks cannot exist on their own but form hadrons. Hadrons that contain an odd number of quarks are called baryons and those that contain an even number are called mesons. Two baryons, the proton and the neutron, make up most of the mass of ordinary matter. Mesons are unstable and the longest-lived last for only a few hundredths of a microsecond. They occur after collisions between particles made of quarks, such as fast-moving protons and neutrons in cosmic rays. Mesons are also produced in cyclotrons or other particle accelerators.

Particles have corresponding antiparticles with the same mass but with opposite electric charges. For example, the antiparticle of the electron is the positron (also known as an anti-electron). The electron has a negative electric charge, the positron has a positive charge. These antiparticles can theoretically form a corresponding form of matter call antimatter. Some particles, such as the photon, are their own antiparticle.

These elementary particles are excitations of the quantum fields that also govern their interactions. The dominant theory explaining these fundamental particles and fields, along with their dynamics, is called the Standard Model. The reconci1iation of gravity to the current particle physics theory is not solved; many theories have addressed this problem, such as loop quantum gravity, string theory and supersymmetry theory.

Practical particle physics is the study of these particles in radioactive processes and in particle accelerators such as the Large Hadron Collider. Theoretical particle physics is the study of these particles in the context of cosmology and quantum theory. The two are closely interrelated: the Higgs boson was postulated by theoretical particle physicists and its presence confirmed by practical experiments.

6.2 STANDARD MODEL

The current state of the classification of all elementary particles is explained by the Standard Model, which gained widespread acceptance in the mid-1970s after experimental confirmation of the existence of quarks. It describes the strong, weak, and electromagnetic fundamental interactions, using mediating gauge bosons. The species of gauge bosons are eight gluons, W^-, W^+ and Z bosons and the photon. The Standard Model also contains 24 fundamental fermions (12 particles and their associated anti-particles), which are the constituents of all matter. Finally, the Standard Model also predicted the existence of a type of boson known as the Higgs boson. On 4 July 2012, physicists with the Large Hadron Collider at CERN announced they had found a new particle that behaves similarly to what is expected from the Higgs boson.

The Standard Model, as currently formulated, has 61 elementary particles. Those elementary particles can combine to form composite particles, accounting for the hundreds of other species of particles that have been discovered till date. The Standard Model has been found to agree with almost all the experimental tests conducted to date. However, most particle physicists believe that it is an incomplete description of nature and that a more fundamental theory awaits discovery. In recent years, measurements of neutrino mass have provided the first experimental deviations from the Standard Model, since neutrinos do not have mass in the Standard Model.

6.3 SUBATOMIC PARTICLES

Modern particle physics research is focused on subatomic particles, including atomic constituents, such as electrons, protons, and neutrons (protons and neutrons are composite particles called baryons, made of quarks), that are produced by radioactive and scattering processes; such particles are photons, neutrinos, and muons as well as a wide range of exotic particles. All particles and their interactions observed to date can be described almost entirely by the Standard Model.

Elementary Particles

	Types	Generations	Antiparticle	Colours	Total
Quarks	2	3	Pair	3	36
Leptons			Pair	None	12
Gluons	1	None	Own	8	8
Photon			Own	None	1
Z Boson			Own		1
W Boson			Pair		2
Higgs			Own		1
Total number of (known) elementary particles:					61

Dynamics of particles are also governed by quantum mechanics; they exhibit wave-particle duality, displaying particle-like behaviour under certain experimental conditions and wave-like behaviour in others. In more technical terms, they are described by quantum state vectors in a Hilbert space, which is also treated in quantum field theory. Following the convention of particle physicists, the term *elementary particles* is applied to those particles that are, according to current understanding, presumed to be indivisible and not composed of other particles.

Bosons

Bosons are the mediators or carriers of fundamental interactions, such as electromagnetism, the weak interaction, and the strong interaction. Electromagnetism is mediated by the photon, the quanta of light. The weak interaction is mediated by the W and Z bosons. The strong interaction is mediated by the gluon, which can link quarks together to form composite particles. Due to the aforementioned color confinement, gluons are never observed independently. The Higgs boson gives mass to the W and Z bosons via the Higgs mechanism – the gluon and photon are expected to be massless. All bosons have an integer quantum spin (0 and 1) and can have the same quantum state.

Antiparticles and Color Charge

Most aforementioned particles have corresponding antiparticles, which compose antimatter. Normal particles have positive lepton or baryon number, and antiparticles have these numbers negative. Most properties of corresponding antiparticles and particles are the same, with a few gets reversed; the electron's antiparticle, positron, has an opposite charge. To differentiate between antiparticles and particles, a plus or negative sign is added in superscript. For example, the electron and the positron are denoted e^- and e^+. When a particle and an antiparticle interacts with each other, they are annihilated and convert to other particles. Some particles have no antiparticles, such as the photon or gluon.

Quarks and gluons additionally have color charges, which influences the strong interaction. Quark's color charges are called red, green and blue (though the particle itself have no physical color), and in antiquarks are called antired, antigreen and antiblue. The gluon can have eight color changes, which are the result of quarks' interactions to form composite particles (gauge symmetry SU(3)).

6.4 INADEQUACY OF CLASSICAL MECHANICS

The development of classical mechanics is based on Newton's three laws (*i*) the law of inertia (*ii*) law of force (*iii*) the law of action and reaction. These laws include the concept of absolute mass, absolute space and absolute time. The classical mechanics explains correctly the motion of celestial bodies like planets, stars and macroscopic as well as microscopic terrestrial bodies moving with non-relativistic speeds (*i.e.*, $v << c$, c being the speed of light in vacuum).

(*i*) The necessity for a departure from classical mechanics is clearly shown by experimental results.

The forces known in classical electrodynamics are inadequate for the explanation of the remarkable stability of atoms and molecules, which is necessary in order that materials may have definite physical and chemical properties at all.

(*ii*) It does not hold in the region of atomic dimensions *i.e.,* it cannot explain the non-relativistic motion of atoms, electrons, protons etc.

(*iii*) It could not explain observed spectrum of black body radiations, the observed variation of specific heat of metals and gases.

(*iv*) It could not explain the origin of discrete spectra of atoms since according to classical mechanics the energy changes are always continuous. This difficulty was later on resolved by Bohr to some extent.

In spite of this classical mechanics could not explain a large number of observed phenomenon like photoelectric effect, Compton effect, Raman effect, etc.

6.5 BLACK BODY RADIATION

A perfectly black body is one which absorbs totally all the radiation of any wavelength which fall on it. Such a body does not reflect any radiation and so it appears black. Experimentally, such a body is represented by a hollow container with a small hole in the wall. When such a body is heated, it emits radiations of all possible wavelengths. These radiations are independent of the nature of the substance. Such heat radiations in a uniform temperature enclosure are known as **black body radiations**. Lummer and Pringsheim (1899) made experiments to determine the distribution of energy among radiations of different wavelengths emitted by a black body at various temperatures. Fig. 6.1 shows the variation of intensity of radiation with the wavelengths at different temperatures. A close investigation reveals the following important facts.

(*i*) At a given temperature, the energy is not uniformly distributed in the radiation spectrum.

(*ii*) At a given temperature, the intensity of radiation increases with increase in wavelength and becomes maximum at a particular wavelength. With further increase in wavelengths the intensity of radiation decreases.

(*iii*) An increase in temperature causes a decrease in λ_m the wavelength for which energy emitted is maximum.

(*iv*) For all wavelengths, an increase in temperature causes an increase in the energy emission.

(*v*) The area under each curve represents the total energy emitted for the complete spectrum at a particular temperature.

(*a*) In order to explain the observed spectra by applying the classical thermodynamics, it was shown by Wein that the amount of energy contained in the spectral region included with in the wavelength λ and $\lambda + d\lambda$ emitted by a black body is given by,

$$E_\lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{C}{\lambda T}} . d\lambda \qquad \ldots(1)$$

where A and C are constants.

This formula works well only for short wavelengths, but did not reproduce the results at long wavelengths and high temperatures. Equation (1) gives finite energy even for T = ∞. Lord Rayleigh argued that it is, unlikely that E should be finite for infinite value of temperature.

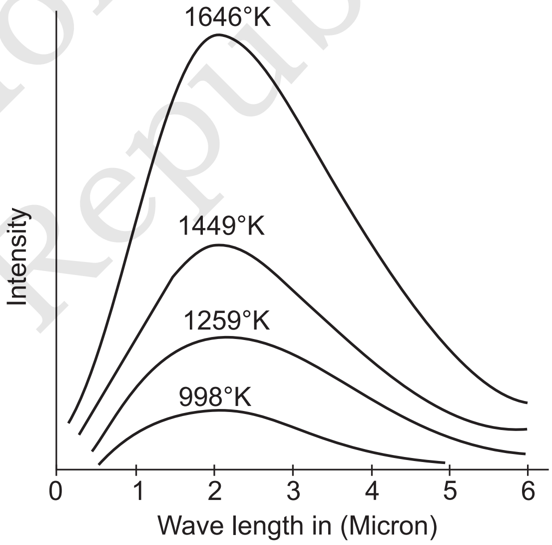


Fig. 6.1

(*b*) **Rayleigh and Jeans,** by assuming that the radiation in black body have degrees of freedom and applying the law of equipartition of energy showed that

$$E_\lambda d\lambda = \frac{A}{\lambda^4} T d\lambda \qquad \ldots(2)$$

where A is a constant.

It is clear from this formula that the energy radiated in a given wavelength range $d\lambda$ increases rapidly as λ decreases and approaches infinity for very short wavelengths, which cannot be true. Thus the formula (2) holds good in the region of longer wavelengths but fails for the shorter wavelengths. Thus, Wien's law and Rayleigh Jean's law do not precisely agree with the experimental results.

Planck's Radiation Law

In order to explain the spectrum of black body radiation, Planck put forward his quantum hypothesis, according to which a black body contains simple harmonic oscillators which are capable of vibrating with all possible frequencies. The frequency of a radiation emitted by an oscillator is the same as the frequency of its vibration. An oscillator cannot emit energy in a continuous manner, it can emit energy in the multiples of the unit called quantum. If an oscillator is vibrating with a frequency ν, it can only radiate in quanta of magnitude $h\nu$, where h is a constant, called Planck's constant and its value is 6.625×10^{-27} erg-sec.

If N is the total number of Planck's oscillator and E is their total energy, then the average energy per oscillator is given by,

$$\bar{\in} = \frac{E}{N} \qquad \ldots(3)$$

Let N_0, N_1, N_2, ..., N_n etc. be the number of oscillators having energies 0, $\in = h\nu$, $2\in$, ..., $n\in$ etc. respectively, then

$$N = N_0 + N_1 + \ldots + N_n = \sum_{n=0}^{\infty} N_n \qquad \ldots(4)$$

and

$$E = \in[N_1 + 2N_2 + \ldots + nN_n + \ldots) = \sum_{n=0}^{\infty} n\in N_n \qquad \ldots(5)$$

According to Maxwell's distribution formula, the probability for an oscillator to possess an energy E is given by,

$$\text{Exp}\,[-E/kT]$$

Hence the average energy per oscillator can be written as

$$\bar{\in} = \frac{E}{N} = \frac{\sum_{n=0}^{\infty} n\in e^{-\frac{n\in}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{n\in}{kT}}} \qquad \ldots(6)$$

Simplifying we get,

$$\bar{\in} = \frac{h\nu}{e^{h\nu}kT - 1} \qquad \ldots(7)$$

Now, the number of oscillators per unit volume in the frequency range ν and ν + *d*ν is given by,

$$N = \frac{8\pi\nu^2 d\nu}{e^3} \quad ...(8)$$

Multiplying it by the average energy per oscillator, given by eqn. (7), we get the total energy per unit volume belonging to the range *d*ν or the energy density belonging to range *d*ν as

$$\boxed{E_\nu d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT} - 1}d\nu} \quad ...(9)$$

This is known as Planck's radiation law.

Planck's radiation law explains all the observed facts of the black body spectrum for the entire wavelength range.

Above description illustrates that Planck's law is perfect one and all other laws follow as special cases of this law, which account only for limited portion of the black body spectrum.

6.6 QUANTUM THEORY OF RADIATION AND PHOTON

The quantum theory of radiation was proposed by Max Planck to explain the distribution of energy with wavelength in black body radiation. He put forward the hypothesis of atomicity of energy and introduced his **quantum** of action. This theory, has been able to explain a number of phenomenona concerning the interaction of energy with matter. According to this theory, an accelerated electron, which Planck described as a **linear oscillator** does not radiate energy continuously as required by the electromagnetic theory of light but the energy is emitted in tiny packets or **quanta.**

The oscillator emit energy only when it passes from a higher energy state to lower energy state and absorbs energy when it goes from a lower energy state to a higher energy state. No emission or absorption of energy takes place when the oscillator is in a given state. The smallest amount of energy which can be emitted or absorbed by the oscillator is *h*ν. In other words a radiation of frequency ν is emitted as quantum of energy E = *h*ν where *h* is Planck's universal constant of action having a value equal to 6.62×10^{-34} J sec. The quantum is the basic unit of energy and cannot be subdivided. It is the atom of energy and is known as **Photon.**

This theory has been able to explain a number of phenomena which could not be explained according to the classical concept of radiation. For example, it gives a very satisfactory explanation of variation of specific heat of solids with temperature, photo electric effect, Compton effect etc. and has been successfully applied by Bohr in the theory of hydrogen spectrum.

(*a*) **Energy of photon.** Energy of photon is only in multiples of $h\nu$, where h is Planck's constant and ν its frequency. If a photon undergoes interaction with matter, either it can be completely absorbed, transferring all its energy or it may transfer part of its energy, and its frequency is adjusted to a lower value thereby, maintaining its particle character. If many photons exist, they have more energy and intensity of radiation is also large. It means that intensity is not concerned with the individual photon energy but simply gives their number. Hence, energy is only dependent on the intrinsic property of the photons. Thus the energy of photon is independent of its intensity, depending only on its frequency, a concept contrary to classical ideas where radiation is considered purely as waves and energy estimated by the intensity of the wave disturbance, dependent on the physical properties of the medium.

(*b*) **Mass and momentum of photon.** As photons have energy (E = $h\nu$) and are in motion with velocity c. According to the theory of relativity a mass m has an energy equivalent to mc^2.

Hence $$E = mc^2$$

$$m = E/c^2$$

$$\text{Mass of photon} = h\nu/c^2 \qquad \text{...(10)}$$

$$\text{As momentum} = \text{mass} \times \text{velocity}$$

$$= \frac{h\nu}{c^2} \times c = \frac{h\nu}{c} = \frac{h}{\lambda} \qquad \text{...(11)}$$

where λ is the wavelength of the radiation.

A photon is a partide of zero rest mass. Its dynamical variables are energy ($h\nu$) and momentum (h/λ). A photon has zero charge and spin equal to one quantum unit $\left(\frac{h}{2\pi} = \hbar\right)$. Two charges or two magnetic poles exert a force on each other due to exchange of photons. A photon interacts with all the charged particles and can also interact with some neutral particles.

A photon behaves as a small elastic sphere for all purposes. When a photon collides with an electron both momentum and energy are conserved as in the case of collision between two electric spheres. This fact has been verified by Compton in his experiments on X-rays. When X-rays of frequency ν collide with an electron of mass *m*, then

$$h\nu = \frac{1}{2}mv^2 + h\nu' \qquad ...(12)$$

where v is the velocity with which electron moves and ν′ is the frequency of the new radiation given out.

(*c*) Non-electrical nature of photons. The photons constituting radiations are electrically neutral. They are not affected by electric or magnetic fields and also do not ionise directly by themselves. However, they can eject charged particles from matter, when they impinge on atoms.

6.7 PHOTO-ELECTRIC EFFECT

Liberation of electrons from matter under the influence of sufficiently high frequency electromagnetic radiations is known as photo-electric effect and emitted electrons, are called photo-electrons.

Experimental Arrangement

The experimental arrangement to demonstrate photo-electric effect is shown in Fig. 6.2

It consists of an evacuated glass tube T fitted with a photo sensitive plate A and a quartz window W for allowing the ultra violet light to fall on the plate A from a source S. Another plate B is also sealed at the other end of the tube. The middle point of the H.T battery and the plate A are earth connected, so that the potential of plate A is zero. The plate B can be given a positive or a negative potential by the potential dividing arrangement as shown in Fig. 6.2.

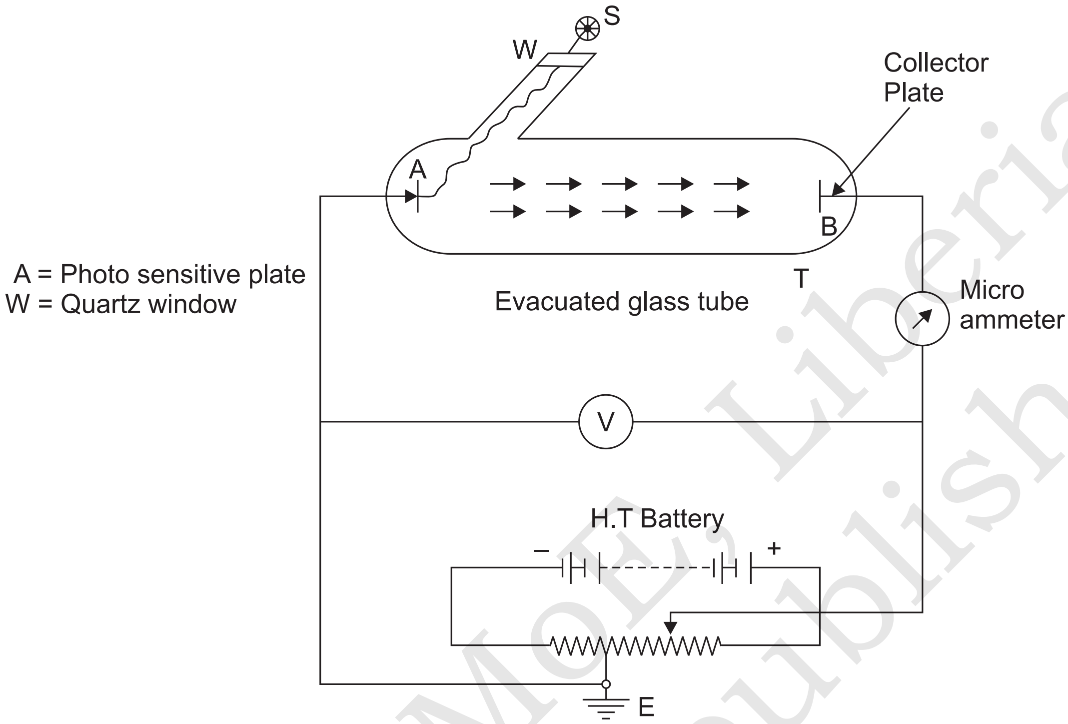


Fig. 6.2

When light of suitable frequency is allowed to fall on the plate A from a source S through the quartz window W and the plate B is kept at a positive potential an electric current flows in the outer circuit as indicated by the micro-ammeter. This is due to photo electrons emitted from the plate A which are attracted towards the plate B as shown. Some important results are as under.

(*a*) Intensity effect. For a given metal rate of emission of photo-electrons *i.e.*, photo current is directly proportional to the intensity of incident radiation for a given light Fig. 6.3 provided the frequency is above the threshold frequency *i.e.,* a bright light always gives more photo current than a dim one for a given frequency.

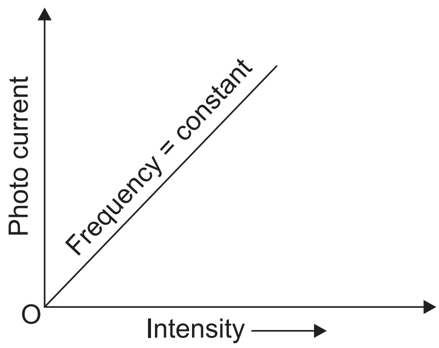


Fig. 6.3

(*b*) Frequency effect. For a given metal maximum kinetic energy of photoelectrons varies linearly with the frequency of incident radiation provided it is greater than the threshold frequency and is independent of its intensity *i.e.,* blue light will always give more energetic photo electrons than red, what ever be its intensity.

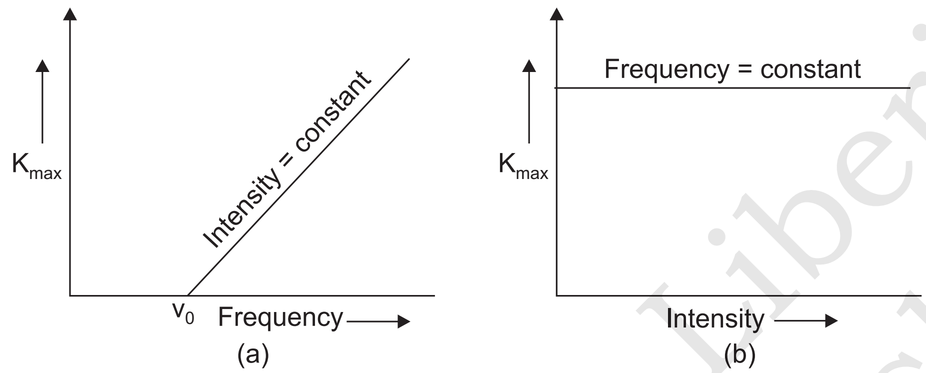


Fig. 6.4

(*c*) Effect of nature of metal. If light of different frequencies in turn is incident on a given metal, photoelectric effect takes place only if the frequency of incident radiation is more (or wavelength is less) than a specific value ν_0. This specific value of frequency (ν_0) is called threshold frequency or cutoff frequency and depends only on the nature of metal.

(*d*) Time-delay effect. Within the limits of experimental accuracy (about 10^{-9} sec), there is no time log between incidence of radiation and emission of photoelectrons *i.e.,* as soon as light is incident on the metal, photo electrons are emitted.

6.8 FAILURE OF (CLASSICAL) ELECTROMAGNETIC THEORY

According to the electromagnetic theory of light, a substance exposed to light is subjected to an oscillatory electric field, the intensity of light being proportional to the square of the amplitude of electric vector. The electrons within the substance interact with the field, gain kinetic energy, and leave the substance as soon as the kinetic energy exceeds the binding energy. The electromagnetic energy gained by the electron depends upon

(*i*) intensity of incident light.

(*ii*) area of the surface over which light falls

(*iii*) effective area of the electron exposed to light and

(*iv*) time for which the surface is illuminated.

If we consider the light falling on a sodium surface, a detectable photo-electric current will be obtained when 10^{-6} W/m^2 of electromagnetic energy is absorbed by the surface. A layer of sodium one atom thick and 1 m^2 in area has about 10^{19} atoms. If we suppose that the incident light is absorbed by the uppermost layer of sodium atoms, each atom will receive an average amount of energy equal to 10^{-25} W so that 1.6×10^6 sec will be required by an atom to have 1 eV of energy and an electron in sodium will gain sufficient energy to be liberated in about 2 months.

Hence the electromagnetic theory fails to explain there is no time lag between the instant the light falls and that of the emission of photo electrons.

Secondly, according to classical electromagnetic theory, the ejection of an electron should depend upon the incident energy *i.e.* , intensity of light and not on frequency. In other words there should be no threshold frequency for any material. This is contrary to the observed facts.

Again, there is no limit on the maximum energy to be transferred to the electron *i.e.*, the electron can have any value of maximum energy. This is contrary to observed facts. Hence classical electromagnetic theory of light fails to explain the basic facts of photo electric effect.

6.9 EINSTEIN'S PHOTO-ELECTRIC EQUATION

In 1905 Einstein proposed a theory based upon Planck's idea of quanta of energy which gave a satisfactory explanation of the various experimental facts. According to him, monochromatic light of frequency ν consists of photons of energy $h\nu$. When a photon with a sufficient energy content strikes an electron of a photo sensitive material, a part of its energy known as the work function W_0 of the surface, is used up in liberating the electron from the surface, whereas, the remaining is spent in imparting kinetic energy to it. If m is the mass and v is the maximum velocity of the emitted electron, then

$$h\nu = W_0 + \frac{1}{2}mv_{max}^2 \qquad ...(13)$$

This is known as Einstein's photo-electric equation. The maximum kinetic energy of the emitted electron is given from eq. (13) is

$$\frac{1}{2}mv^2 = (h\nu - W_0) \qquad ...(14)$$

It follows from eq. (14) that the maximum velocity of the emitted electron depends upon the frequency of the incident radiation. An increase in the frequency of the incident light increases the amount of energy carried by each individual light quantum so that, during each collision with a free electron in the metal, these quanta impart larger amount of kinetic energy. An increase in the intensity of light cannot cause any change in the kinetic energy (or the maximum velocity) of the emitted electrons. More intense light simply more number of light quanta having the same energy falling on the surface per sec. Since one light quantum can emit only one electron, the number of electrons emitted will correspondingly increase.

Further, if ν_0 is the lowest or threshold frequency which just causes the emission of electrons, then we have

$$h\nu_o = W_0$$

therefore eq. (14) reduces to

$$\boxed{\frac{1}{2}mv^2 = h(\nu - \nu_0)} \qquad ...(15)$$

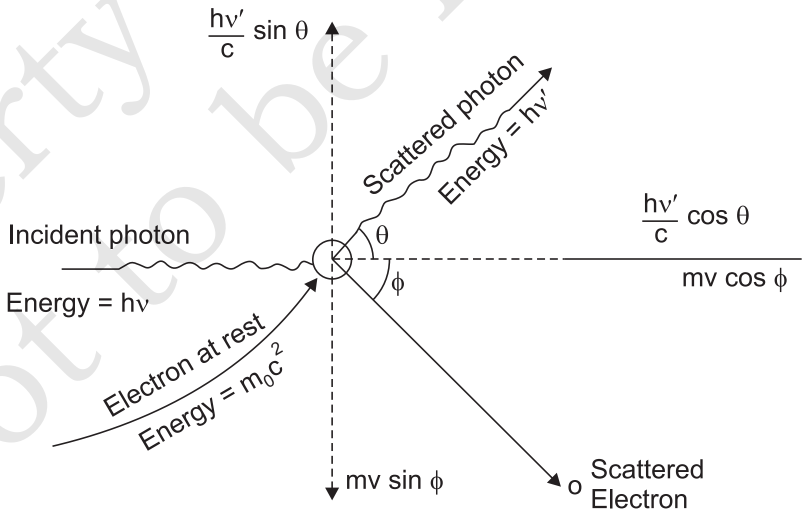


Fig. 6.6

Let us consider the case before the collision

(*i*) Energy of incident photon = $h\nu$

(*ii*) Momentum of incident photon = $\dfrac{h\nu}{c}$

(*iii*) Rest mass of electron = m_0, so that rest mass energy = m_0c^2

(*iv*) Rest mass momentum of electron = 0.

And after the collision

(*i*) Energy of scattered photon = $h\nu'$

(*ii*) Momentum of scattered photon = $\dfrac{h\nu'}{c}$ ...(16)

(*iii*) Mass of the electron moving with velocity $v = m$, so the relativistic energy of electron = mc^2

(*iv*) Momentum of the recoil electron = mv ...(17)

Now applying the principle of conservation of energy, before and after collision, we have

$$h\nu + m_0c^2 = h\nu' + mc^2 \quad ...(18)$$

Again applying the principle of conservation of momentum along the direction of incident photon, before and after the collision, we have

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \quad ...(19)$$

and conservation of momentum ν a direction ⊥ to the direction of incident photon, we have

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \quad ...(20)$$

Now from eq. (19), we have

$$h\nu = h\nu'\cos\theta + mvc\cos\phi$$

or
$$mvc\cos\phi = h\nu - h\nu'\cos\theta \quad ...(21)$$

and From eq. (20), we have

$$mvc\sin\phi = h\nu'\sin\theta \quad ...(22)$$

Squaring (21) and (22) and adding, we get

$$m^2v^2c^2 = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$

or
$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta + h^2\nu'^2\sin^2\theta$$

$$= h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta$$

$$= h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta) \quad ...(23)$$

Also from eq. (18), we have

$$mc^2 = h(\nu - \nu') + m_0c^2$$

squaring $$m^2c^4 = h^2(\nu^2 + \nu'^2 - 2\nu'\nu) + m_0^2c^4 + 2hm_0c^2(\nu - \nu') \quad \ldots(24)$$

Subtracting (23) from (24), we have

or $$m^2c^2(c^2 - v^2) = 2h^2\nu'\nu(\cos\theta - 1) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$
$$= -2h^2\nu'\nu(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

or $$\frac{m_0^2}{(1 - v^2/c^2)} \times c^2(c^2 - v^2) = 2h^2\nu'\nu(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$\left[\because \quad m = \frac{\mu_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}\right]$$

or $$m_0^2c^4 = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

or $$2h^2\nu\nu'(1 - \cos\theta) = 2h(\nu - \nu')m_0c^2$$

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

or $$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\theta) \quad \ldots(25)$$

which may also be written as

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1 - \cos\theta)$$

or $$\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)$$

$$= \frac{h}{m_0c} \cdot 2\sin^2\theta/2 \quad \ldots(26)$$

From eq. (26) it is clear that

(*i*) Change in wavelength is independent of the wavelength of the incident photon as well as the nature of the scattering substance but depends only on the angle of scattered photon.

(*ii*) When $\theta = 0$, $\lambda' - \lambda = 0$, *i.e.*, no scattering takes place along the direction of incident photon.

(*iii*) When $\theta = \pi/2$,

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0c} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 0.024 \text{ Å}$$

This change in wavelength is called Compton's wavelength is denoted λ_c and as is clear from above it is constant.

(*iv*) When $\theta = \pi$,

$$\lambda - \lambda' = \Delta\lambda = \frac{2h}{m_0 c} = 0.48 \text{ Å}$$

So when θ varies between 0° and π, the wavelength of scattered photon varies between λ to $\lambda + \frac{2h}{m_0 c}$, provided the incident photon is of very small wavelength.

Direction of the recoil electron. Dividing eq. (22) by eq. (21), we have

$$\tan\phi = \frac{h\nu' \sin\theta}{h\nu - h\nu' \cos\theta} = \frac{\nu' \sin\theta}{\nu - \nu' \cos\theta} \qquad \text{...(27)}$$

Also from eq. (25), we have

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1 - \cos\theta)$$

or $$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2} \cdot 2\sin^2\frac{\theta}{2}$$

or $$\frac{\nu}{\nu'} = 1 + \frac{h\nu}{m_0 c^2} \cdot 2\sin^2\frac{\theta}{2}$$

or $$\nu' = \frac{\nu}{1 + \left(\frac{h\nu}{m_0 c^2}\right) \times 2\sin^2\frac{\theta}{2}}$$

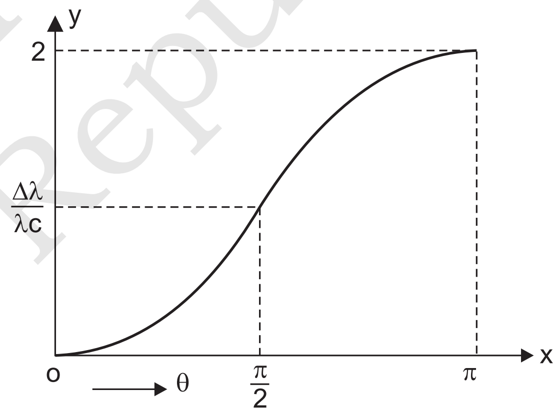


Fig. 6.7

Putting this value of ν' in equation (27), we get

$$\tan\phi = \frac{\dfrac{\nu \sin\theta}{\left(1 + \alpha \cdot 2\sin^2\frac{\theta}{2}\right)}}{\nu - \left[\dfrac{\nu}{\left(1 + \alpha \cdot 2\sin^2\frac{\theta}{2}\right)}\right]\cos\theta}$$

where $\alpha = \frac{h\nu}{m_0 c^2}$

$$\therefore \qquad \tan\phi = \frac{\sin\theta}{1 + 2\alpha\sin^2\frac{\theta}{2} - \cos\theta} = \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{1 + 2\alpha\sin^2\frac{\theta}{2} - \cos\theta}$$

$$= \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{(1-\cos\theta) + 2\alpha\sin^2\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} + 2\alpha\sin^2\frac{\theta}{2}}$$

$$\frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}(1+\alpha)} = \frac{\cot\frac{\theta}{2}}{1+\alpha}$$

or

$$\tan\phi = \frac{\cot\frac{\theta}{2}}{\left(1 + \frac{h\nu}{m_0c^2}\right)} \qquad \text{...(28)}$$

which gives the direction of the recoil electron in terms of the frequency of the incident photon and the direction of the scattered photon.

Behaviour of photon and electron when θ varies from 0 to π

(*i*) When $\theta = 0$, $\cot\frac{\theta}{2} = \infty$

$$\therefore \qquad \tan\phi = \infty \quad \text{or} \quad \phi = \frac{\theta}{2}$$

Also $\Delta\lambda = 0$

This means that the photon goes unscattered, while the electron moves at right angles to the direction of the photon.

(*ii*) When $\theta = \frac{\pi}{2}$; $\cot\frac{\theta}{2} = 1$; $\tan\phi = \frac{1}{1+\alpha}$

This being positive ϕ lies between 0 and 90°

$$\Delta\lambda = \frac{h}{m_0c} = 0.0242 \text{ Å}$$

This means that photon gets scattered and moves at right angles to the direction of incidence. The change in wavelength is 0.0242 Å while the electron moves in any direction making an angle ϕ less than $\frac{\pi}{2}$ with the direction of incidence.

(*iii*) When $\theta = \pi$; $\cot \frac{\theta}{2} = 0$, $\therefore$ $\tan \phi = 0$, or $\phi = 0$

and $$\Delta\lambda = \frac{2h}{m_0 c} = 0.0484 \text{ Å}$$

This means that the photon reverses its direction with change in wavelength of 0.0484 Å while the electron moves in the direction of the incident photon. Thus, maximum change in wavelength take place when the photon gets scattered in the direction of incidence.

(*iv*) **Electron cannot be scattered at greater than 90° ·**

The angle of scattering for the electron ϕ is related to the angle of scattering for the photon θ as under.

$$\tan \phi = \frac{\cot \frac{\theta}{2}}{1 + \frac{h\nu}{m_0 c^2}}$$

Now $$1 + \frac{h\nu}{m_0 c^2} = a \text{ constant} = k$$

$$\tan \phi = k \cot \frac{\theta}{2}$$

The maximum value of $\cot \frac{\theta}{2} = \infty$.

Hence the maximum value of $\tan \phi = \infty$ or $\phi = \frac{\pi}{2}$.

In other words the electron cannot be scattered at angle greater than 90°.

Kinetic Energy of Recoil Electron

In the compton effect the K.E. of the recoil electron will be equal to the decrease in the energy of the incident photon.

Let the energy of incident photon = $h\nu$, and that of scattered photon = $h\nu'$.

So decrease in energy of photon = $h\nu - h\nu'$. Let mc^2 be the energy of the recoil electron and m_0c^2 be the rest mass energy of the electron.

$\therefore$ K.E. of recoil electron = $(m - m_0)c^2$

which corresponds to the decrease in the energy of the incident photon

∴ K.E. of the recoil electron

$$E = h(\nu - \nu') = h\nu\left(1 - \frac{\nu'}{\nu}\right) = h\nu\left(1 - \frac{c\lambda}{c\lambda'}\right)$$

$$= h\nu\left(1 - \frac{\lambda}{\lambda'}\right) = h\nu\left(\frac{\lambda' - \lambda}{\lambda'}\right) = h\nu\frac{\Delta\lambda}{\lambda + \Delta\lambda}$$

Now $\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$

∴ K.E. of recoil electron

$$E = \frac{h\nu \times \frac{h}{m_0 c}(1 - \cos\theta)}{\lambda + \frac{h}{m_0 c}(1 - \cos\theta)}$$

$$= \frac{h^2\nu}{m_0 c\lambda}\frac{(1 - \cos\theta)}{1 + \frac{h}{m_0 c\lambda}(1 - \cos\theta)} \quad \text{[Dividing by } \lambda]$$

$$= \frac{\frac{h^2\nu^2}{m_0 c^2}(1 - \cos\theta)}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)} \quad [\because \ c = \nu\lambda]$$

which shows that K.E. of the recoil electron depends upon the scattering angle θ.

(*i*) When θ = 0, E = 0

i.e., the electron will not recoil if the scattering photon has scattering angle θ = 0.

(*ii*) When θ = π/2

$$E = \frac{\frac{h^2\nu^2}{m_0 c^2}}{1 + \frac{h\nu}{m_0 c^2}} = \frac{h\nu\alpha}{1 + \alpha}$$

(where $\alpha = h\nu/m_0 c^2$)

(*iii*) When θ = π,

$$E = \frac{\frac{2h^2\nu^2}{m_0 c^2}}{1 + \frac{2h\nu}{m_0 c^2}} = \frac{2h\nu\alpha}{1 + 2\alpha} = \frac{h\nu(2\alpha)}{1 + 2\alpha}$$

So we see that

(a) where $\theta = 0$, E = 0 *i .e.,* minimum of energy is transferred to recoil electron.

(b) where $\theta = \pi$, $E = h\nu \times \frac{2\alpha}{1+2\alpha}$ *i.e.,* maximum of energy is transferred to electron.

However $\frac{2\alpha}{1+2\alpha} < 1$,

which means that the energy transferred to the electron is less than the energy of incident photon, *i.e., photon cannot transfer whole of energy to the electron in photo-electric effec,t.*

Experimental Set up of Compton Effect

Fig. 6.7 shows the experimental set up for the study of Compton effect. A monochromatic beam of X-rays is allowed to fall on a carbon block B which acts as a scatterer. Normally Compton effect can be best observed in case of elements of low atomic number because the binding energies of the electrons, in these elements are negligible as compared to the quantum energy of the incident photon. The scattering of X-ray photons can take place in different directions and their intensities and wavelengths can be measured by Bragg's spectrometer.

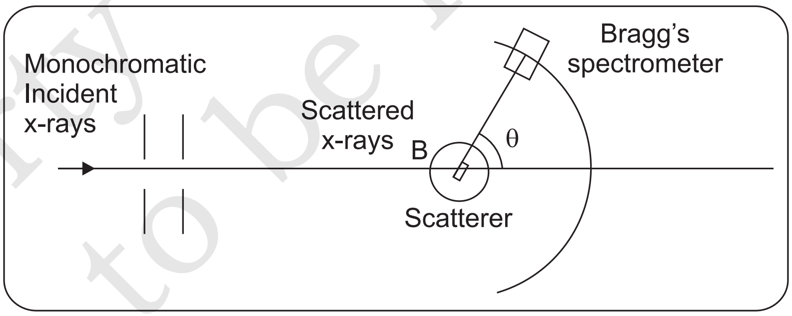


Fig. 6.7

In a typical experiment, K_α line of molybdenum was scattered by gyaphite and the distribution of intensities was studied at different angles. If a graph is plotted between intensities and wavelengths, then we observe that for each value of θ, there are distinct intensity peaks for two wavelengths, one of which corresponds to the incident radiation λ and the other has a higher value λ'. The peak corresponding to λ' is called modified peak.

We can see that with increase of θ, $\lambda' - \lambda$ increases and the shift in wavelength $\Delta\lambda$ increases in accordance with the results obtained by Compton and so the Compton effect is verified experimentally.

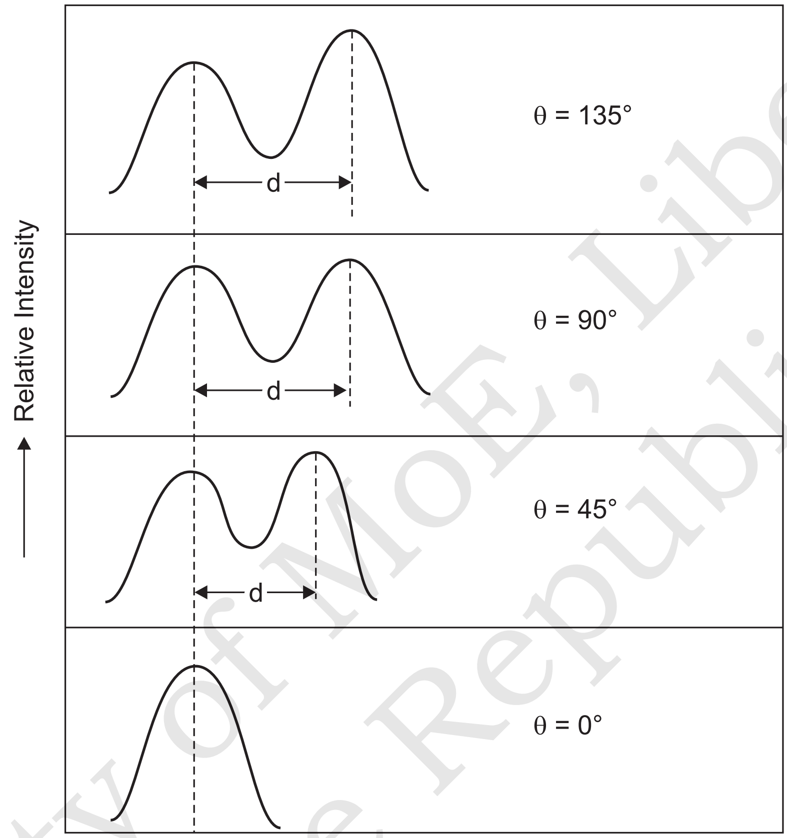


Fig. 6.8

Example 1: *Show that the energy lost by a photon in Compton interaction with a free stationary electron can be written as* $h\nu\left[\dfrac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}\right]$

where $\alpha = \dfrac{h\nu}{m_0c^2}$.

Solution: We have the change in wavelength given by relation

$$\lambda' - \lambda = \frac{h\nu}{m_0 c}(1 - \cos\theta) \qquad ...(29)$$

Also, when a photon of energy $h\nu$ is scattered by a free electron, it imparts some energy to the electron and scatters with less energy $h\nu'$. The loss of energy is given by

$$h\nu - h\nu' = h(\nu - \nu') = h\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right)$$

$$= hc\frac{(\lambda' - \lambda)}{\lambda\lambda'} \qquad \text{...(30)}$$

Putting the value of λ' from (29) in (30), we get loss of energy as

$$\text{Energy lost} = \frac{hc\left[\frac{h}{m_0 c}(1 - \cos\theta)\right]}{\lambda\left[\lambda + \frac{h}{m_0 c}(1 - \cos\theta)\right]}$$

$$= \frac{h^2}{m_0}\left[\frac{(1 - \cos\theta)}{\lambda^2\left[1 + \frac{h}{m_0 c\lambda}(1 - \cos\theta)\right]}\right]$$

$$= \frac{h^2\nu^2}{m_0 c^2}\left[\frac{(1 - \cos\theta)}{\left[1 + \frac{h}{m_0 c\lambda}(1 - \cos\theta)\right]}\right]$$

$$= h\nu \times \frac{h\nu}{m_0 c^2} \times \left[\frac{1 - \cos\theta}{1 + \frac{h}{m_0 c^2}(1 - \cos\theta)}\right]$$

$$= h\nu\left[\frac{\alpha(1 + \cos\theta)}{1 + \alpha(1 - \cos\theta)}\right]$$

Example 2: *X-ray with λ = 1 Å are scattered from a carbon block. The scattered radiation is viewed at 90° to the incident beam. (a) What is the Compton shift $\Delta\lambda$? What K.E. is imparted to the recoiling electron?*

Solution: (*a*) We have the Compton shift given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

In this case, $\theta = 90°$, $\cos 90° = 0$

$$\Delta\lambda = \frac{h}{m_0 c} = \frac{6.62 \times 10^{-34}\ \text{Js}}{9.1 \times 10^{-31}\ \text{kg} \times 3 \times 10^{8}\ \text{ms}^{-1}}$$

$$= 2.43 \times 10^{-12}\ \text{m}$$

= 0.0243 Å

(*b*) From the conservation law of energy, we have

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \text{K}$$

where K is the K.E. of recoil electron,

Also, $\lambda' = \lambda + \Delta\lambda$

$$\therefore \quad \frac{hc}{\lambda} = \frac{hc}{\lambda + \Delta\lambda} + \text{K} \quad \text{or} \quad \text{K} = \frac{hc\Delta\lambda}{\lambda(\lambda + \Delta\lambda)}$$

$$\text{K} = \frac{6.62 \times 13^{-34}\ \text{Js} \times 3 \times 10^{8}\ \text{ms}^{-1} \times 2.43 \times 10^{-12}\ \text{m}}{1 \times 10^{-10}\ \text{m} \times (1 + 0.0243) \times 10^{-10}\ \text{m}}$$

$$= 4.72 \times 10^{-17}\ \text{J}$$

$$= \frac{4.72 \times 10^{-17}}{1.6 \times 10^{-19}}\text{eV} = \mathbf{.295\ eV}$$

Example 3: *Find the wavelength of X-ray photons which produce recoil electrons of energy* 5 *keV in Compton effect. Assume the direction of recoil electron to be in the direction of incident photon while photon is scattered through an angle of 180°.*

Solution: In this case direction of scattered photon $\phi = 180°$

Also, $\dfrac{hc}{\lambda} - \dfrac{hc}{\lambda'} = 5\ \text{keV}$

$$= 5000 \times 1.6 \times 10^{-19}\ \text{J}$$

$$\therefore \quad \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 8 \times 10^{16}\ \text{J} \qquad \text{...(31)}$$

From conservation law of momentum in the direction of incident photon,

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos 180° + mv \cos 0°$$

$$= \frac{h}{\lambda'}\cos 180° + mv$$

or $$\frac{h}{\lambda} = \frac{h}{\lambda'} \times -1 + mv$$

$$\therefore \quad \frac{h}{\lambda} + \frac{h}{\lambda'} = \sqrt{\frac{1}{2}mv^2 \times 2m}$$

$$= \sqrt{8 \times 10^{16} \times 2 \times 91 \times 10^{-31}} = 38.1 \times 10^{-24}$$

Multiplying both sides by c,

$$\Rightarrow \qquad \frac{hc}{\lambda} + \frac{hc}{\lambda'} = 3 \times 10^{8} \times 38.1 \times 10^{-24}$$

$$= 11.448 \times 10^{-17} \text{ J} \qquad \ldots(32)$$

Adding (31) and (32), we get

$$\frac{2hc}{\lambda} = 8 \times 10^{-16} + 11.448 \times 10^{-17}$$

$$= 122.48 \times 10^{-16} \text{ J}$$

or

$$\lambda = \frac{2hc}{122.48 \times 10^{-16} \text{ J}}$$

$$= (2 \times 6.6 \times 10^{-34} \text{ J}) \times \frac{(3 \times 10^{8} \text{ m/s})}{122.48 \times 10^{-16} \text{ J}}$$

$$= 0.324 \times 10^{-10} \text{ m}$$

$$= \mathbf{0.324\ Å}$$

6.10 SHORTCOMINGS OF OLD QUANTUM THEORY

The quantum theory based on Bohr's quantum condition and Wilson Sommerfield quantization rule for periodic systems could explain only certain limited problems like energy state of hydrogenation, particle in box, harmonic oscillator, rigid rotator etc. Main shortcomings of old quantum theory are:

(*i*) It could not be applied to non-periodic systems.

(*ii*) It could not explain the spectral lines of relatively simple cases like hydrogen molecule and normal helium atom.

(*iii*) It could not explain the relative intensities of spectral lines at all.

(*iv*) It could not explain process connected with the spin of electrons and Pauli's exclusion principle.

(*v*) Bohr's postulate of discrete, non-radiating energy states on which old quantum theory is based, was empirical without having any theoretical ground. The above difficulties have been resolved by the development of a new method of approach–"Wave Mechanics."

In wave mechanics, Schrodinger by using the concept given by de Broglie derived an equation called Schrodinger's equation which is able to explain the complete behaviour of atomic system in many cases.

Further, half integral values and integral values are obtained automatically during the mathematical solution of the equation. Later on, this was refined by Heisenberg, Born, Jordan, Dirac and others, thereby finally leading to development of new quantum theory.

6.11 UNCERTAINTY PRINCIPLE

The reconciliation of the corpuscular nature with the wave character of light (and also of the electron) has been brought about through the modern quantum theory; and perhaps the best known consequence of wave-particle duality is the uncertainty principle of Heisenberg which ‘ can be stated as follows:

If the x-coordinate of the position of a particle is *known to an accuracy* Δx*, then the x-component of the momentum cannot be determined to an accuracy better than* $\Delta p_x \approx h/\Delta x$*, where h is the Planck's constant.*

Alternatively, one can say that if Δx and Δp_x represent the accuracies with which the *x*-coordinate of the position and the *x*-component of the momentum can be determined, then the following inequality must be satisfied

$$\Delta x \Delta p_x \geq h \qquad ...(30)$$

We do not feel the effect of this inequality in our everyday experience because of the smallness of the value of Planck's constant ($\approx 6.6 \times 10^{-27}$ erg-sec). For example, for a tiny particle of mass 10^{-6} g, if the position is determined within an accuracy of about 10^{-6} cm, then according to the uncertainty principle, its velocity cannot be determined within an accuracy better than $\Delta v = 6 \times 10^{-16}$ cm/sec. This value is much smaller than the accuracies with which one can determine the velocity of the particle. For a particle of a greater mass, Δv will be even smaller. Indeed, had the value of Planck's constant been much larger, the world would have been totally different.

REVIEW EXERCISE

A. MULTIPLE CHOICE QUESTIONS (MCQs)

1. Choose the wrong statement about the spin of an electron, according to quantum mechanics:

(a) It is related to intrinsic angular momentum.

(b) Spin is the rotation of an electron about its own axis.

(c) Value of the spin quantum number must not be 1.

(d) +1/2 value of spin quantum number represents up spin.

2. The Quantum Mechanical Model of the atom was proposed by:

(a) Louis de Broglie (b) Erwin Schrodinger

(c) Neil Bohr (d) Werner Heisenberg

3. The wavelength of the matter waves is independent of:

(a) mass (b) velocity (c) charge (d) momentum

4. Assuming the velocity to be the same, which particle is having longest wavelength

(a) an electron (b) a proton (c) a neutron (d) an α-particle

5. The uncertainty principle states that the error in measurement is due to:

(a) dual nature of particles

(b) due to the small size of particles

(c) due to large size of particles

(d) due to error in measuring instrument

6. The Eigen value of a particle in a box is ___________

(a) $\frac{L}{2}$ (b) $\frac{2}{L}$ (c) $\sqrt{\frac{L}{2}}$ (d) $\sqrt{\frac{2}{L}}$

7. Particle in a box can never be at rest.

(a) True (b) False

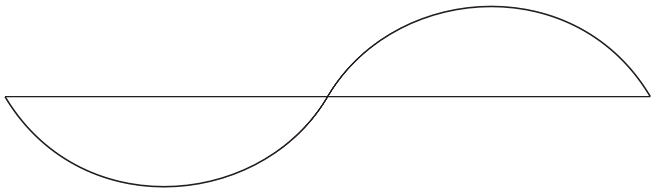
8. What is the minimum Energy possessed by the particle in a box?

(a) Zero (b) $\frac{\pi^2\hbar^2}{2mL^2}$ (c) $\frac{\pi^2\hbar^2}{2mL}$ (d) $\frac{\pi^2\hbar}{2mL}$

9. The wave function of a particle in a box is given by ____________

(a) $\sqrt{\frac{2}{L}}\sin\frac{nx}{L}$ (b) $\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ (c) $\sqrt{\frac{2}{L}}\sin\frac{x}{L}$ (d) $\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}$

10. The wave function for which quantum state is shown in the figure?



(a) 1 (b) 2 (c) 3 (d)

B. FILL IN THE BLANKS

1. The walls of a particle in a box are supposed to be ___________

2. The wave function of the particle lies in which region ___________?

3. The particle loses energy when it collides with the wall ___________.

4. The Energy of the particle is proportional to _________

5. For a particle inside a box, the potential is maximum at x = __________

C. VERY SHORT ANSWER TYPE QUESTIONS

1. If the uncertainty in the velocity of a moving object is 1.0×10^{-6} ms^{-1} and the uncertainty in its positions is 58 m, the mass of this object is approximately equal to that of:

2. Define the law of Stefan-Boltzmann.

3. Mention the uses of an electron microscope.

4. Explain Planck's hypothesis or what are the postulates of Planck's quantum theory?

5. What is the wavelength associated with a photon of a light with the energy is 3.6×10^{-19} J?

D. SHORT ANSWER TYPE QUESTIONS

1. Describe Planck's three experimental observations that explain the photoelectric effect.

2. What is the difference between classical and quantum mechanics? Provide the equation relating the energy of emitted radiation to frequency.

3. State Heisenberg's uncertainty principle. Give its mathematical expression.

4. An electron and a photon each have a wavelength of 1.00 nm. Find

(i) their momentum,

(ii) the energy of the photon, and

(iii) the kinetic energy of the electron.

5. Find the uncertainty in the position of an electron when the mass of an electron is 9.1×10^{-28} g and the uncertainty in velocity is equal to 2×10^{-3} cm/sec.

E. LONG ANSWER TYPE QUESTIONS

1. A photon of energy 1.02 MeV is scattered through 90° by a free electron. Calculate the energy ot photon and electron after interaction.
2. When ultraviolet radiation of wavelength 1200 Å. is incident on a photo sensitive surface, the electrons emitted have a stopping potential of 5.6 volt. Calculate the work function, threshold frequency and cut off wavelength.
3. On increasing the wavelength of the incident radiation from λ_1 to λ_2 the stopping potential of the photoelectrons emitted is changed by V. Calculate *h*, Planck's constant.
4. Electrons are emitted with zero velocity from a certain metal surface when it is exposed to radiation of wavelength λ = 6800 Å. Calculate threshold frequency and work function of metal.
5. A photoelectric surface has a work function of 4 eV. What is the maximum velocity of photoelectrons emitted by light of frequency 10^{15} Hertz incident on the surface.